Référentiels et bases associées

Résumé

Projections

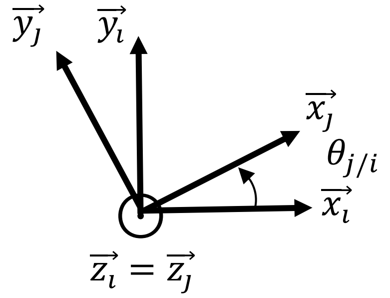
Le minimum vital

Programme - Compétences		
B29	MODELISER	Solide indéformable: - référentiel, repère - vecteur-vitesse angulaire de deux référentiels en mouvement l'un par rapport à l'autre

Les projections – Le minimum vital

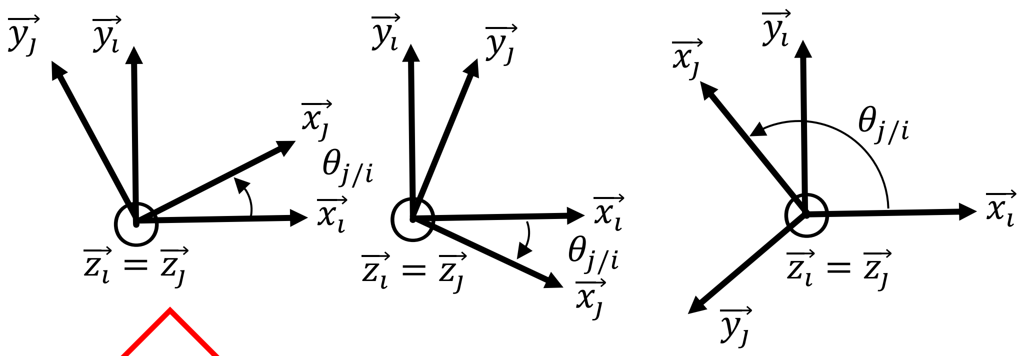
Paramétrage angulaire

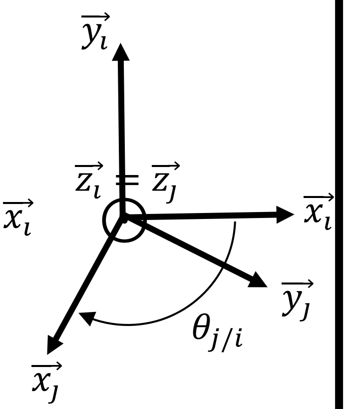
Un angle $\theta_{j/i}$ est un angle orienté, algébrique, allant de $\vec{x_i}$ vers $\vec{x_j}$
Il est représenté par une flèche allant de $\vec{x_i}$ vers $\vec{x_j}$ dont le sens définit le signe (sens direct

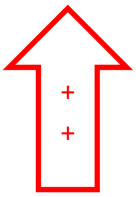


$$\theta_{j/i} = \widehat{(\vec{x_i}, \vec{x_j})}[2\pi] = \widehat{(\vec{y_i}, \vec{y_j})}[2\pi]$$

Projections / Rotation $\vec{z}$



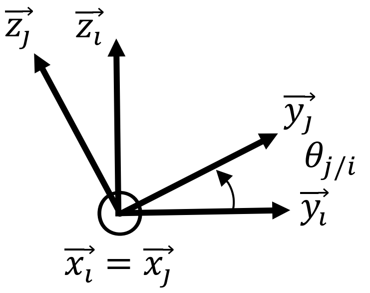




$$\forall (i,j) \in \mathbb{N}^2$$
$$\vec{x_j} = \cos\theta_{j/i}\,\vec{x_i} + \sin\theta_{j/i}\,\vec{y_i}$$
$$\vec{y_j} = -\sin\theta_{j/i}\,\vec{x_i} + \cos\theta_{j/i}\,\vec{y_i}$$

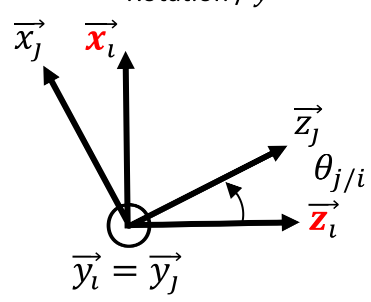
3 rotations usuelles

Rotation / $\vec{x}$



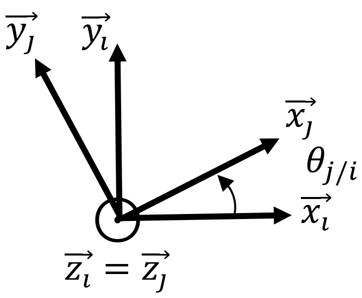
$$\vec{y_j} = \cos\theta_{j/i}\,\vec{y_i} + \sin\theta_{j/i}\,\vec{z_i}$$
$$\vec{z_j} = -\sin\theta_{j/i}\,\vec{y_i} + \cos\theta_{j/i}\,\vec{z_i}$$

Rotation / $\vec{y}$



$$\vec{z_j} = \cos\theta_{j/i}\,\vec{z_i} + \sin\theta_{j/i}\,\vec{x_i}$$
$$\vec{x_j} = -\sin\theta_{j/i}\,\vec{z_i} + \cos\theta_{j/i}\,\vec{x_i}$$

Rotation / $\vec{z}$

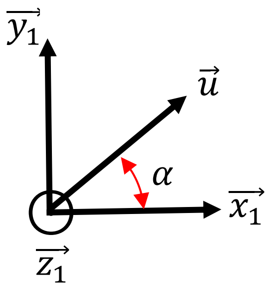


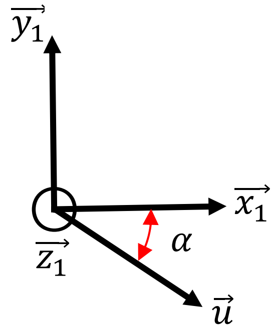
$$\vec{x_j} = \cos\theta_{j/i}\,\vec{x_i} + \sin\theta_{j/i}\,\vec{y_i}$$
$$\vec{y_j} = -\sin\theta_{j/i}\,\vec{x_i} + \cos\theta_{j/i}\,\vec{y_i}$$

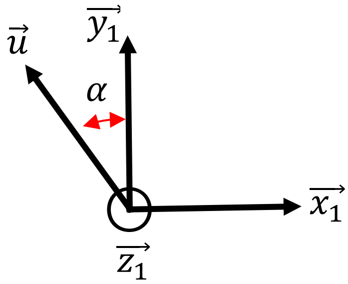
Astuce : Vecteur rotation « vers nous » et représentation directe de la base

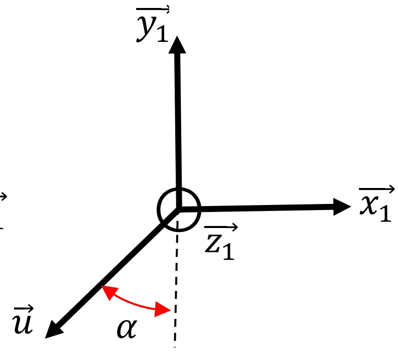
Cas des angles non orientés - Exemples

$\alpha \in \left[0; \frac{\pi}{2}\right]$









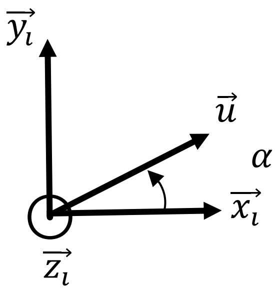
$\vec{u} = \cos\alpha\,\vec{x_1} + \sin\alpha\,\vec{y_1}$ $\quad \vec{u} = \cos\alpha\,\vec{x_1} - \sin\alpha\,\vec{y_1}$ $\quad \vec{u} = -\sin\alpha\,\vec{x_1} + \cos\alpha\,\vec{y_1}$ $\quad \vec{u} = -\sin\alpha\,\vec{x_1} - \cos\alpha\,\vec{y_1}$

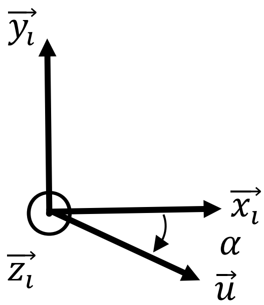
Si $\alpha \in \left[0; \frac{\pi}{2}\right]$, on peut « lire visuellement » les projections sur les axes de la base de projection

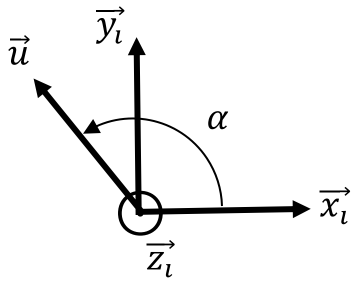
Si $\alpha \notin \left[0; \frac{\pi}{2}\right]$, se rapporter au cas « Angles orientés & non orientés » en y associant un angle orienté

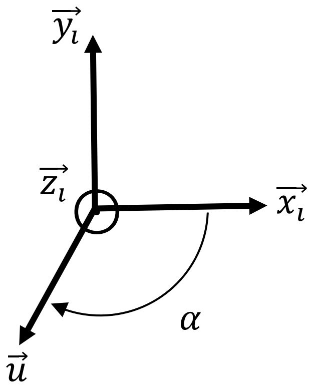
Projection d'un vecteur quelconque

Orientation / $\vec{x_\iota}$



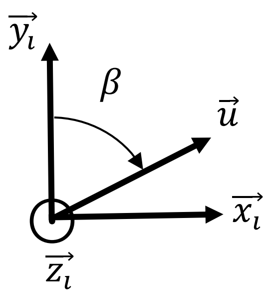


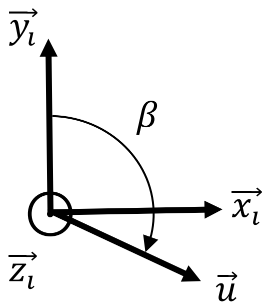


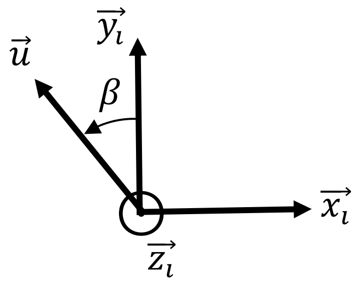


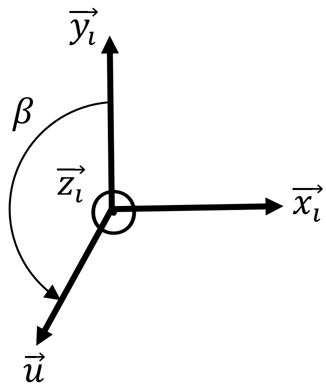
$$\boldsymbol{\vec{u} = \cos\alpha\,\vec{x_\iota} + \sin\alpha\,\vec{y_\iota}}$$

Orientation / $\vec{y_\iota}$



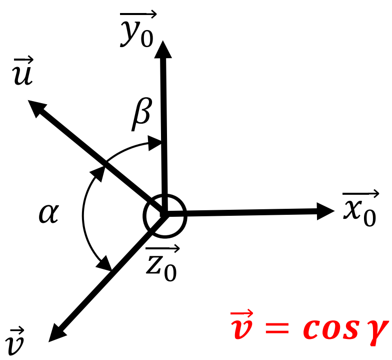




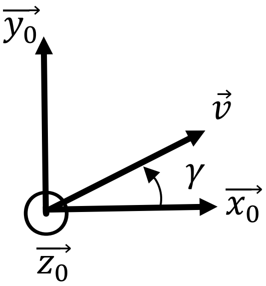


$$\boldsymbol{\vec{u} = -\sin\beta\,\vec{x_\iota} + \cos\beta\,\vec{y_\iota}}$$

Angles orientés & non orientés - Exemple



$\Leftrightarrow$



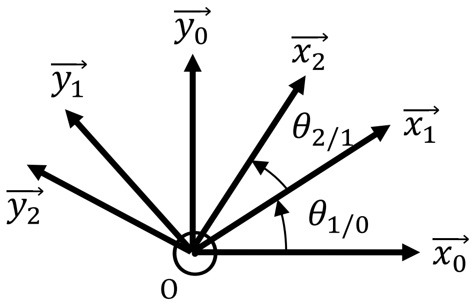
$$\vec{v} = \cos\gamma\,\vec{x_0} + \sin\gamma\,\vec{y_0}$$

$$\gamma = (\widehat{\vec{x_0},\vec{v}}) = (\widehat{\vec{x_0},\vec{y_0}}) + (\widehat{\vec{y_0},\vec{u}}) + (\widehat{\vec{u},\vec{v}}) = (\alpha - \beta) + \frac{\pi}{2}$$

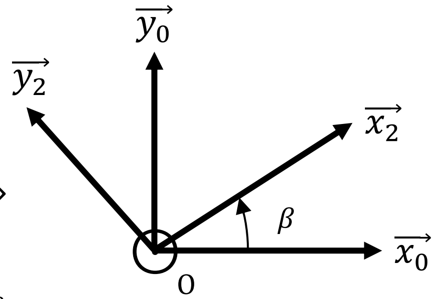
Relation de Chasles angulaire

Angle non orientés associés au signe correspondant au sens de leur parcours

Changements de bases successifs



$\Leftrightarrow$



$$\beta = \theta_{2/1} + \theta_{1/0}$$
$$\vec{x_2} = cos(\theta_{2/1} + \theta_{1/0})\vec{x_0} + sin(\theta_{2/1} + \theta_{1/0})\vec{y_0}$$
$$\vec{y_2} = -sin(\theta_{2/1} + \theta_{1/0})\vec{x_0} + cos(\theta_{2/1} + \theta_{1/0})\vec{y_0}$$

Ne pas projeter dans $\mathfrak{B}_1$ puis dans $\mathfrak{B}_0$